L6 Assessment 1 Revision Questions Solutions

1.

(i)	$(-4)^2 - 4 \times k \times k$ $= 16 - 4k^2$	M1 A1 2	Uses $b^2 - 4ac$ (involving k) $16 - 4k^2$
(ii)	$16 - 4k^2 = 0$ $k^2 = 4$ $k = 2$ or $k = -2$	M1 B1 B1 3 **5**	Attempts $b^2 - 4ac = 0$ (involving k) or attempts to complete square (involving k)

2.

$(-30)^2 - 4 \times k \times 25k = 0$	**M1**	Attempts $b^2 - 4ac$ involving k
$900 - 100k^2 = 0$	**M1**	States their discriminant = 0
$k = 3$	**B1**	
or $k = -3$	**B1** **4**	
	4	

3.

$y = x^2 - 4x + 5$
$y = m + 2x - x^2$

$\Rightarrow x^2 - 4x + 5 = m + 2x - x^2$

$\Rightarrow 2x^2 - 6x + (5 - m) = 0$

LOOKING FOR REPEATED ROOTS

$b^2 - 4ac = 0$

$\Rightarrow (-6)^2 - 4 \times 2 \times (5 - m) = 0$

$\Rightarrow 36 - 8(5 - m) = 0$

$\Rightarrow 36 = 8(5 - m)$

$\Rightarrow \frac{9}{2} = 5 - m$

$\Rightarrow \boxed{m = \frac{1}{2}}$

If $m = \frac{1}{2}$

$\Rightarrow 2x^2 - 6x + (5 - m) = 0$

$\Rightarrow 2x^2 - 6x + (5 - \frac{1}{2}) = 0$

$\Rightarrow 2x^2 - 6x + \frac{9}{2} = 0$

$\Rightarrow 4x^2 - 12x + 9 = 0$

$\Rightarrow (2x - 3)^2 = 0$

$\boxed{x = \frac{3}{2}}$

$\Rightarrow y = x^2 - 4x + 5$

$\Rightarrow y = \frac{9}{4} - 4(\frac{3}{2}) + 5$

$\Rightarrow y = \frac{9}{4} - 6 + 5$

$\boxed{y = \frac{5}{4}}$

$\therefore P(\frac{3}{2}, \frac{5}{4})$

4.

$y = x^2 - 2x - 8$

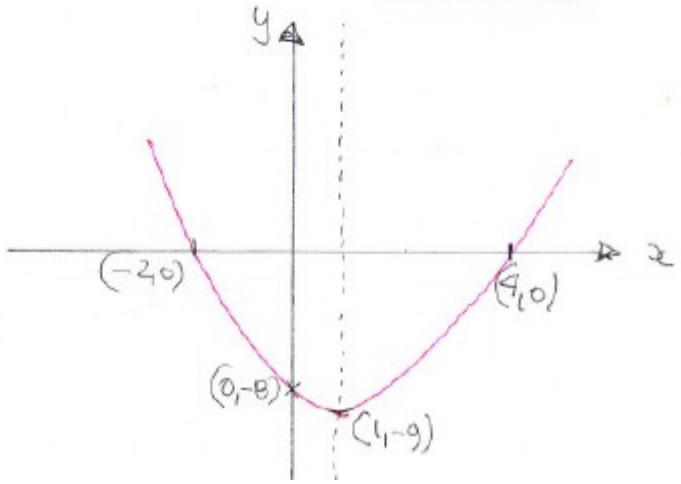
- +ve $x^2 \Longrightarrow$ ∪
- When $x=0$, $y=-8 \Longrightarrow (0,-8)$
- When $y=0$, $0 = x^2 - 2x - 8$

  $0 = (x-4)(x+2)$

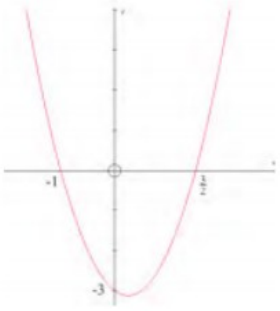
  $x = 4, -2 \Longrightarrow (4,0), (-2,0)$
- $y = x^2 - 2x - 8$

  $y = (x-1)^2 - 1^2 - 8$

  $y = (x-1)^2 - 9 \Longrightarrow$ ∪ with vertex $(1,-9)$



5.

(i)		$(2x-3)(x+1)=0$ $x=\frac{3}{2}, x=-1$ 	M1 A1 A1**ft** B1 **[4]**	Correct method to find roots – **see appendix 1** Correct roots Good curve: • Correct shape, symmetrical positive quadratic • Minimum point in the correct quadrant for their roots **(ft)** • their x intercepts correctly labelled **(ft)** y intercept at (0, −3). Must have a graph.
(ii)		$x<-1, x>\frac{3}{2}$	M1 A1**ft** **[2]**	Chooses the "outside region" Follow through x-values in (i). Allow "$x<-1, x>\frac{3}{2}$", "$x<-1$ **or** $x>\frac{3}{2}$" but do not allow "$x<-1$ **and** $x>\frac{3}{2}$"
(iii)		$b^2-4ac=1^2-4\times 2\times -(3+k)$ $25+8k<0$ $k<-\frac{25}{8}$	M1 A1 A1 **[3]**	Rearrangement and use of $b^2-4ac<0$, must involve 3 and k in constant term (not $3k$) $p+8k<0$ oe found, any constant p. p need not be simplified Correct final answer

6.

(a) (I) $f(x) = (x-4-\sqrt{3})(x-4+\sqrt{3})$

$f(x) = x^2 - 4x + \sqrt{3}x$
$\quad -4x \quad +16 - 4\sqrt{3}$
$\quad -\sqrt{3}x - 3 + 4\sqrt{3}$

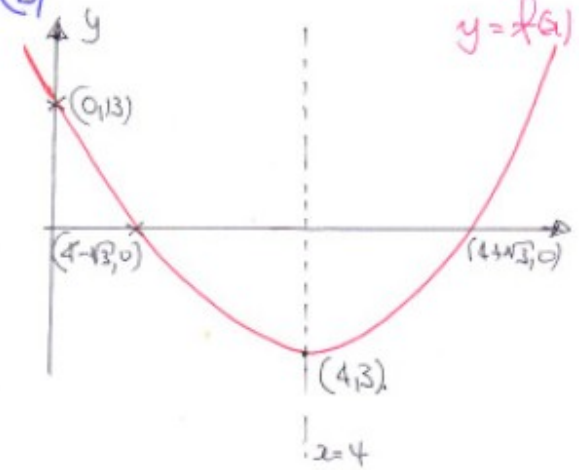
$f(x) = x^2 - 8x + 13$

(II) $f(x) = x^2 - 8x + 13$

$f(x) = (x-4)^2 - 16 + 13$

$f(x) = (x-4)^2 - 3$

(b)



when $y=0$ $\quad (x-4-\sqrt{3})(x-4+\sqrt{3}) = 0$

$[x-(4+\sqrt{3})][x-(4-\sqrt{3})] = 0$

$x = 4+\sqrt{3}$ or $4-\sqrt{3}$

7.

(a) $f(x) = x^2 - 2x - 47$

$f(x) = (x-1)^2 - 1 - 47$

$f(x) = (x-1)^2 - 48$

(b) $f(x) = 0$

$\Rightarrow (x-1)^2 - 48 = 0$

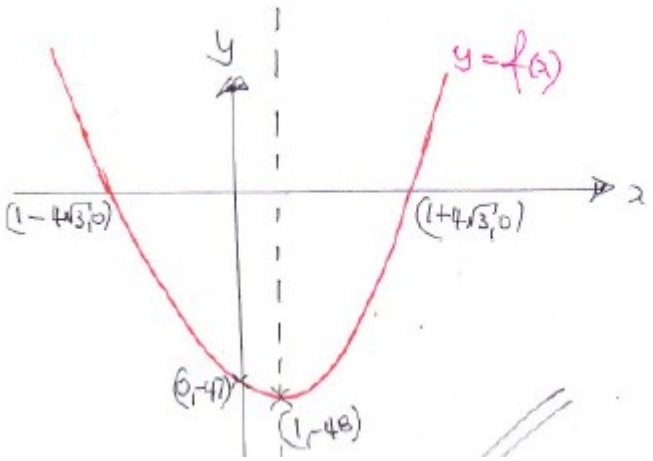
$\Rightarrow (x-1)^2 = 48$

$\Rightarrow x - 1 = \pm\sqrt{48}$

$\Rightarrow x - 1 = \pm 4\sqrt{3}$

$\Rightarrow x = 1 \pm 4\sqrt{3}$

(c)



8.

(a) $f(x) = 4x^2 + 4x - 1$

$f(x) = 4\left[x^2 + x - \frac{1}{4}\right]$

$f(x) = 4\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{1}{4}\right]$

$f(x) = 4\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{2}\right]$

$f(x) = 4\left(x + \frac{1}{2}\right)^2 - 2$

(b) $4\left(x + \frac{1}{2}\right)^2 - 2 = 0$

$\Rightarrow 4\left(x + \frac{1}{2}\right)^2 = 2$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 = \frac{1}{2}$

$\Rightarrow x + \frac{1}{2} = \pm\sqrt{\frac{1}{2}}$

$\Rightarrow x = -\frac{1}{2} \pm \frac{\sqrt{2}}{2}$

$\Rightarrow x = \frac{-1 \pm \sqrt{2}}{2}$

9.

Let $y = x^{\frac{1}{3}}$ $y^2 + 3y - 10 = 0$ $(y-2)(y+5) = 0$ $y = 2, y = -5$ $x = 2^3, x = (-5)^3$ $x = 8, x = -125$	*M1 DM1 A1 DM1 A1 ft 5 **5**	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct attempt to solve quadratic Both values correct Attempt cube Both answers correctly followed through **SR** B2 $x = 8$ from T & I

10.

$y = x^{\frac{1}{2}}$

$2y^2 - 7y + 3 = 0$

$(2y-1)(y-3) = 0$

$y = \frac{1}{2}, y = 3$

$x = \frac{1}{4}, x = 9$

M1* Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{2}}$

M1dep Correct method to solve a quadratic

A1

M1 Attempt to square to obtain x

A1

SR If first M1 not gained and 3 and ½ given as final answers, award B1

5

11.

Let $y = x^{\frac{1}{3}}$ $3y^2 + y - 2 = 0$ $(3y-2)(y+1) = 0$ $y = \frac{2}{3}, y = -1$ $x = \left(\frac{2}{3}\right)^3, x = (-1)^3$ $x = \frac{8}{27}, x = -1$	*M1 DM1 A1 DM1 A1 ft 5 **5**	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct method to find roots Both values correct Attempt cube of at least one value Both answers correctly followed through **SR** If M1* not awarded, **B1** x = -1 from T & I

12.

$$\frac{x}{x-2} + 4 = \frac{3}{x} \quad \text{MULTIPLY BY } x$$

$$\Rightarrow \frac{x^2}{x-2} + 4x = 3 \quad \text{MULTIPLY BY } x-2$$

$$\Rightarrow x^2 + 4x(x-2) = 3(x-2)$$

$$\Rightarrow x^2 + 4x^2 - 8x = 3x - 6$$

$$\Rightarrow 5x^2 - 11x + 6 = 0$$

$$\Rightarrow (5x-6)(x-1) = 0$$

$$\therefore x = 1, \frac{6}{5}$$

13.

$$\frac{2}{x-3} + \frac{13}{x^2+4x-21} = 1$$

$$\Rightarrow \frac{2}{x-3} + \frac{13}{(x-3)(x+7)} = 1$$

$$\Rightarrow \frac{2(x+7)+13}{(x-3)(x+7)} = 1$$

$$\Rightarrow \frac{2x+14+13}{x^2+4x-21} = 1$$

$$\Rightarrow 2x+27 = x^2+4x-21$$

$$\Rightarrow 0 = x^2+2x-48$$

$$\Rightarrow (x+8)(x-6) = 0$$

$$\Rightarrow x = -8, 6$$

14.

$$\frac{x^2+3x}{x^2+5x+6} = \frac{2x^2-x-1}{x^2+8x-9}$$

$$\Rightarrow \frac{x(x+3)}{(x+3)(x+2)} = \frac{(2x+1)(x-1)}{(x-1)(x+9)}$$

$$\Rightarrow \frac{x}{x+2} = \frac{2x+1}{x+9}$$

$$\Rightarrow x(x+9) = (x+2)(2x+1)$$

$$\Rightarrow x^2+9x = 2x^2+5x+2$$

$$\Rightarrow 0 = x^2-4x+2$$

$\Rightarrow$ BY COMPLETING THE SQUARE

$$\Rightarrow 0 = (x-2)^2-4+2$$

$$\Rightarrow 0 = (x-2)^2-2$$

$$\Rightarrow 2 = (x-2)^2$$

$$\Rightarrow \pm\sqrt{2} = x-2$$

$$\Rightarrow x = 2 \pm \sqrt{2}$$

15.

(b) $\frac{\sqrt{9x^6y^4}}{(3x^2y^3)^2} = \frac{3x^3y^2}{9x^4y^6} = \frac{1}{3}x^{-1}y^{-4}$ or $\frac{1}{3xy^4}$

16.

(b) $\sqrt{\frac{3a^3bc \times 6a^2b^2c^3}{2abc^4}} = \sqrt{\frac{18a^5b^3c^4}{2abc^4}} = \sqrt{9a^4b^2} = 3a^2b$

17.

(b) $\left(k^{\frac{3}{2}} \times 8k^{-3}\right)^{\frac{1}{3}} = \left(8k^{-\frac{3}{2}}\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(k^{-\frac{3}{2}}\right)^{\frac{1}{3}} = 2 \times k^{-\frac{1}{2}}$

$$= \frac{2}{k^{\frac{1}{2}}} = \frac{2}{\sqrt{k}}$$

18.

a $243\sqrt{3}$

$= 3^5 \times 3^{\frac{1}{2}}$

$= 3^{5+\frac{1}{2}}$

$= 3^{\frac{11}{2}}$

$a = \frac{11}{2}$

b From part a: $3^x \times 27^y = 243\sqrt{3} = 3^{\frac{11}{2}}$

$3^x \times 3^{3y} = 3^{\frac{11}{2}}$

$3^{x+3y} = 3^{\frac{11}{2}}$

So $x + 3y = \frac{11}{2}$

$3y = \frac{11-2x}{2}$

$y = \frac{11-2x}{6}$

19.

$$\frac{\sqrt{98}-\sqrt{8}}{1+\sqrt{2}} = \frac{\sqrt{49}\sqrt{2}-\sqrt{4}\sqrt{2}}{1+\sqrt{2}} = \frac{7\sqrt{2}-2\sqrt{2}}{1+\sqrt{2}} = \frac{5\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{5\sqrt{2}(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{5\sqrt{2}-10}{1-\sqrt{2}+\sqrt{2}-2} = \frac{5\sqrt{2}-10}{-1}$$

$$= 10 - 5\sqrt{2}$$

20.

$A = WL$

$12 = W(2+\sqrt{7})$

$W = \frac{12}{2+\sqrt{7}}$

$\therefore W = \frac{12(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})} = \frac{24-12\sqrt{7}}{4-2\sqrt{7}+2\sqrt{7}-7} = \frac{24-12\sqrt{7}}{-3} = -8+4\sqrt{7}$

21.

(a) $\sqrt{50} + \sqrt{3} \times \sqrt{6} - \frac{14}{\sqrt{2}}$

$= \sqrt{25}\sqrt{2} + \sqrt{18} - \frac{14\sqrt{2}}{\sqrt{2}\sqrt{2}}$

$= 5\sqrt{2} + \sqrt{9}\sqrt{2} - \frac{14\sqrt{2}}{2}$

$= 5\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$

$= \sqrt{2}$

(b) $(\sqrt{75} - \sqrt{48})^2$

$= [\sqrt{25}\sqrt{3} - \sqrt{16}\sqrt{3}]^2$

$= [5\sqrt{3} - 4\sqrt{3}]^2$

$= [\sqrt{3}]^2$

$= 3$